7355 – IS STA – M – 21

FIRST SEMESTER M.A. / M.Sc. (CBCS) DEGREE EXAMINATION, MARCH 2021

STATISTICS

Paper: STCT 1.1

(Linear Algebra)

Time : 3 Hours] [Max. Marks : 75

Answer any **five** *questions including question* **1.**

Each question carries **15** *marks.*

1. a) Define Linearly independent and dependent vectors. If $x_1, x_2, \ldots\ldots x_n$ are linearly independent, then show that the subset of three vectors is also linearly independent.

   b) Define row and column space. State and prove replacement lemma.

   c) Define i) elementary operations, ii) row-reduced echelon matrix and iii) rank of a matrix.

   d) Define g-inverse and show that it is not unique.

   e) Explain the classification of quadratic forms.

2. a) Define i) vector space and vector subspace and ii) basis and norm of a vector space.

   b) Let the set of k vectors $\{x_1, x_2, \ldots\ldots x_k\}$ is linearly independent. Then the set $\{x_1, x_2, \ldots\ldots x_k, y\}$ of vectors is linearly dependent if and only if y can be expressed uniquely as a linear combination of $x_1, x_2, \ldots\ldots x_k$.

   c) The set of n vectors $\{x_1, x_2, \ldots\ldots x_n\}$, where $n > 1$, is a linearly dependent set if at least one vectors in the set can be expressed as a linear combination of the remaining vectors.

   4 + 6 + 5 = 15

3. a) Define inner product and state its properties. State and prove Gram-Schmidt orthonormalization process.

   b) If $x_1 = (1, 1, 1, 1)$, $x_2 = (1, -2, 1, -2)$ and $x_3 = (3, 1, 1, -1)$ are the vectors. Determine the set of linearly independent vectors from these vectors and then construct orthonormal basis for the set linearly independent vectors. 7 + 8 = 15

4. a) Define row nullity, column nullity and nullity of a matrix. State and prove the consistency of a system of equations $Ax = b$.

   b) Let A be a $m \times n$ matrix. Then show that the following properties hold

      i) If B be an $n \times p$ matrix, $\text{rank}(AB) \leq \min(\text{rank}(A), \text{rank}(B))$

      ii) If B be an $m \times n$ matrix, $\text{rank}(A + B) \leq \text{rank}(A) + \text{rank}(B)$. 6 + 9 = 15

5. a) Show that for a system of equations $Ax = 0$, where A is a matrix of order $m \times n$ $(m < n)$ the number of linearly independent solutions is $n - \text{rank}(A)$.

b) Solve the system of equations

$$5x + 3y + 7z = 4$$

$$3x + 2y + 2z = 9$$

$$7x + 2y + 10z = 5$$

Verify using $AA^{-}b = b$, where A^{-} is g-inverse of co-efficient matrix of the system.

8 + 7 = 15

6. a) Define Moore-Penrose inverse of a matrix and state its properties.

b) State and prove the uniqueness property of Moore-Penrose inverse of a matrix.

c) Find a g-inverse of $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 3 & 1 \end{bmatrix}$ 3 + 7 + 5 = 15

7. a) Define characteristic polynomial and characteristic value problem. Show that similar matrices have the same characteristic equation.

b) Define idempotent and orthogonal matrix. Prove that their characteristic roots are respectively 0 & 1 and –1 & + 1.

c) Show that the characteristic vectors corresponding to distinct roots of any matrix are linearly independent. 4 + 5 + 6 = 15

8. a) Let A be a $m \times n$ matrix with characteristic roots $\gamma_1, \gamma_2, \ldots, \gamma_m$. Then prove that

i) $\text{tr}(A) = \sum_{i=1}^{m} \gamma_i$ ii) $|A| = \prod_{j=1}^{m} \gamma_i$

b) Define algebraic and geometric multiplicity of a characteristic root and give examples.

c) State and prove the Cayley-Hamilton theorem. 6 + 3 + 9 = 15

9. a) State and prove the spectral decomposition for real symmetric matrices.

b) Prove that for a symmetric and positive definite matrix A, there exists a non singular matrix C such that $C'AC = 1$.

c) Explain canonical form of quadratic forms. 6 + 6 + 3 = 15

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STATISTICS

Paper: STCT 1.2

(Probability Theory)

Time : 3 Hours] [Max. Marks : 75

Question No.1 is **compulsory**.
Answer any **four** *questions from the remaining.*
Each question carries **15** *marks.*

1. a) Find the limit of the sequence

$$A_n = \left\{ w : 0 \leq w < 2 + \frac{(-1)^n}{n} \right\}, n = 1, 2, 3 \ldots.$$

b) Given a class $\{A_i, i = 1, 2, \ldots. n\}$ of n sets, show that $\exists$ a class $\{B_i, i = 1, 2, \ldots. n\}$ of disjoint sets such that $\bigcup_{i=1}^{n} Ai = \sum_{i=1}^{n} Bi$.

c) Show that the set of discontinuity points of a distribution function is atmost countable.

d) Prove that $|\phi(t) - \phi(t+h)|^2 \leq 2\{1 - R_e(\phi(h))\}$

e) Obtain Markov's WLLN. $5 \times 3 = 15$

2. a) Define limit infimum and limit supremum. Show that $\lim_{n \to \infty} \inf A_n \subseteq \lim_{n \to \infty} \sup A_n$.

b) If A_1 and A_2 are the σ- field of subset of Ω, then show that $A = A_1 \bigcap A_2$ is a σ- field of subset of Ω.

c) Let C_1 be the class of all intervals of the form (a, b), a < b, a, b∈R, let $\sigma(C_1)$ be a σ- field generated by C_1. Show that σ-(C_1) is a Borel field. 5 + 5 +5

3. a) Let $A_n \in \Im e$, $n \geq 1$ be a sequence of sets then prove that $P\left(\bigcup_{n=1}^{\infty} A_n\right) \leq \sum_{n=1}^{\infty} P(A_n)$.

b) If $\{A_n)$ is a monotonic increasing sequence of sets belonging to $\Im e$, then establish

$$\lim_{n \to \infty} \mu(A_n) = \mu\left(\bigcup_{n=1}^{\infty} A_n\right).$$

8 + 7

[P.T.O.

4. a) Obtain Jordan's decomposition of distribution function theorem.

b) Let F_1 and F_2 be two distribution functions. If a and b are non-negative integers whose sum is unity, then show that $aF_1 + bF_2$ is also distribution functions. 8 + 7

5. a) State and prove monotone convergence theorem.

b) Let $X_n \xrightarrow{P} X, Y_n \xrightarrow{P} Y$, then show that

(i) $aX_n \xrightarrow{P} aX$ (ii) $X_n + Y_n \xrightarrow{P} X + Y$

(iii) $X_n Y_n \xrightarrow{P} XY$ (iv) $\dfrac{X_n}{Y_n} \xrightarrow{P} \dfrac{X}{Y}$,

if $P[(Y_n) = 0] = 0 = P[Y = 0]$

(v) $f(x_n) \xrightarrow{P} f(x)$, if (.) is continuous. 10 + 5

6. a) Let X be a random variable with distribution function F and characteristic function $\phi(t)$.

Then show that F will have a density function $f(x) = \dfrac{1}{2\pi}\int_{-\infty}^{\infty} \bar{e}^{\,itx}\,\phi(t)\,dt$.

b) Find the probability density function for the given characteristic function $\phi(t) = e\dfrac{-t^2}{2}$.

9 + 6

7. a) Establish relation between CLT and WLLN.

b) State and prove Borel-Cantelli Lemma. 5 + 10

8. a) Obtain Kolmogorov's SLLN for independent random variables.

b) Prove that Lyapunov's conditions for CLT implies Lindberg conditions for CLT.

9 + 6

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FIRST SEMESTER M.A. / M.Sc. (CBCS) DEGREE EXAMINATION, MARCH 2021

STATISTICS

Paper: STCT 1.3

(Theory of Sampling)

Time : 3 Hours] [Max. Marks : 75

Question No. **1** *is* **compulsory.**
Answer any **four** *questions from the remaining.*
Each question carries **15** *marks.*

1. a) Distinguish between census survey and sample survey.

   b) Explain systematic sampling. How does it differ from Simple Random Sampling (SRS)?

   c) Describe and illustrate Lahiri's method of drawing PPS samples.

   d) Define auxiliary variable. Explain about its role in finite sampling.

   e) Explain various sources of non-sampling errors. 5 × 3

2. a) Explain ordered sampling design and unordered sampling design. Illustrate these designs.

   b) Explain stratified sampling. Show that the estimator under this scheme is unbiased.

   c) Explain different types of allocations under stratified sampling. 7 + 4 + 4

3. a) Define an unbiased estimator under PPSWR. Obtain its variance and estimate of the variance.

   b) Define Desraj's ordered estimator. Show that it is unbiased.

   c) Obtain Murthy's unordered estimator from Desraj's ordered estimates and show that it is unbiased. 7 + 4 + 4

4. a) Define Horvitz-Thompson estimator for population mean. Show that it is unbiased and obtain its variance.

   b) Distinguish between PPS and IPPS procedures. Explain Midzuno scheme of sampling and obtain π_{ij} under this scheme. 8 + 7

5. a) Define ratio estimator of the population mean. Stating the assumptions, establish its optimality.

   b) State regression estimator of the population mean. Show that this estimator is more precise than sample mean and ratio estimators. 8 + 7

[P.T.O.

6. a) Define double sampling. Discuss about its applications.

   b) Outline cluster sampling with equal cluster size and develop basic theory for estimating population mean.

   c) Explain two-stage sampling and discuss about its usefulness. 4 + 5 + 6

7. a) Explain the problem of non-response and Politz-Simmon's technique of dealing with it.

   b) What are randomized response techniques? Explain and illustrate Warner's technique. 8 + 7

8. Write short notes on any **three** :

   a) Random sampling

   b) Rao-Hartley-Cochran procedure

   c) Cluster sampling with unequal clusters

   d) Model for measurement of observational error. 3×5

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FIRST SEMESTER M.A / M.Sc. (CBCS) DEGREE EXAMINATION, MARCH 2021

STATISTICS

Paper: STCT 1.4

(Programming in C and simulation)

Time : 3 Hours] [Max. Marks : 75

Question No.1 is **compulsory**.
Answer any **four** *questions from the remaining.*
Each question carries **15** *marks.*

1. a) What are numeric, character and string constants of C.

   b) Write C-statements to declare upper, middle and least as integers; large and small as reals, name and city as string variables of length 15 each.

   c) Write program code to read 3 integers and print the sum and average of read numbers.

   d) Write about *static* type variables.

   e) What are pointers? Explain with example, how pointers are declared. 5 × 3

2. a) List delimiters and their meaning.

   b) Describe getch() and scan f() functions with examples.

   c) Write various mathematical operators supported by C. Explain about hierarchy of evaluation of mathematical expression. 5 + 5 + 5

3. a) Explain C data type conversions with examples.

   b) Write the syntax of multi-directional conditional control structure.

   c) Write a program to print all numbers divisible by 7 between 1 and 100 6 + 6 + 3

4. a) What are multidimensional arrays? Write a program code to declare and initialize 3 × 3 matrix.

   b) Write a program to sort 50 numbers in ascending order. 5 + 10

5. a) Describe built-in and user defined functions.

   b) Using function, write a program to swipe two numbers. 6 + 9

[P.T.O.

6. a) Explain with example, how pointers are used in functions.

   b) Using a structure, write a program to convert time to railway time. 6 + 9

7. a) What are C-FILEs? Describe different modes of opening a FILE.

   b) Describe Box-Muller method of generation of standard normal random variables. 6 + 9

8. a) Write about strlen(), strcmp(), strcpy() and strcat() functions.

   b) Write a program to generate 500 exponential random variables with parameter 0.5. 8+7
