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1579—II S STA—MAY 2019

SECOND SEMESTER M.A./M.Sc. DEGREE EXAMINATION, MAY/JUNE 2019

(CBCS)

Statistics

STCT 2.1—PROBABILITY DISTRIBUTIONS

Time : Three Hours

Maximum : 75 Marks

Answer **Question No. 1** *and any* **four** *from the remaining* **seven** *questions.*
Each question carries 15 *marks.*

1. (a) If $X_1,....,X_{10}$ are independent and identically distributed (i.i.d.) Poisson random variables (r.vs.) with mean 1, find the distribution of $X_1+...,+X_{10}$, it means and its variance.

   (b) Define mixture of two distributions, give one example.

   (c) If X and Y are i.i.d. standard normal r.vs, find the distribution of $X^2 + Y^2$.

   (d) Define non-central Chi-square distribution giving its probability density function (p.d.f.).

   (e) Find the joint distribution of the second minimum and the second maximum from a random sample of size n from an absolutely continuous distribution with distribution function (d.f.) F and p.d.f. f.

(5 × 3 = 15 marks)

2. (a) Prove or disprove. The Geometric r.v. satisfies lack-of-memory property.

   (b) If X and Y are i.i.d. Geometric r.vs, find the distribution of X + Y.

   (c) Define any *one* form of Bivariate exponential distribution. Write the marginals.

(5 + 5 + 5 = 15 marks)

3. (a) If X and Y are independent r.vs. with X ~ Binomial (m, p) and Y ~ Binomial (n, p), find the conditional distribution of X given X + Y and identify the distribution.

   (b) If X and d.f. F which is continuous, find the d.f. of Y = F (X).

   (c) If X ~ Uniform (0, 1), find the d.f. of $Y = -\ln(1-X)$ and identify the distribution.

(5 + 5 + 5 = 15 marks)

4. (a) If X and Y are i.i.d. standard exponential r.vs, find the d.f. of X + Y and identify the same.

   (b) Obtain semi-inter quartile range for standard double exponential distribution.

   (c) Define Weibull distribution and find its mean and variance if they exist.

(5 + 5 + 5 = 15 marks)

Turn over

5. (a) Define bivariate normal distribution and if (X, Y) has standard bivariate normal distribution, find the conditional distribution of Y given $(X = x)$ and identify the same.

   (b) Define non-central F-distribution.

   (c) Describe logistic distribution as a growth function.

(5 + 5 + 5 = 15 marks)

6. (a) Find mode of log normal distribution.

   (b) What are sampling distribution ? Mention any *one* and its use.

   (c) For Cauchy distribution find points of inflexion.

(5 + 5 + 5 = 15 marks)

7. (a) Derive the density function of $T = \frac{X}{\sqrt{\frac{Y}{n}}}$, where X has a standard normal distribution and Y has Chi-square distribution with n d.f.

   (b) Define order statistics and mention any *one* application of it. Let $(X_1, \ldots, X_n)$ be a random sample from $U(0, \theta)$. Find the distribution of r^{th} order statistic.

(7 + 8 = 15 marks)

8. (a) If $X_{(1)} \leq \ldots \leq X_{(n)}$ denote the order statistics from a random sample from the standard exponential distribution, (i) find the p.d.f. of $X_{(2)} - X_{(1)}$; and (ii) show that $X_{(1)}$ and $X_{(n)} - X_{(1)}$ are independent.

   (b) If X_1, X_2, X_3 are i.i.d. standard exponential r.vs show that

$$X_1 + X_2 + X_3, \frac{X_1 + X_2}{X_1 + X_2 + X_3}, \frac{X_1}{X_1 + X_2} \text{ are independent.}$$

(7 + 8 = 15 marks)

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1580—II S STA—MAY 2019

SECOND SEMESTER M.A./M.Sc. DEGREE EXAMINATION, MAY/JUNE 2019

(CBCS)

Statistics

STCT 2.2—THEORY OF POINT ESTIMATION

Time : Three Hours

Maximum : 75 Marks

Answer any **five** *questions.*
Question No. 1 *is* **Compulsory**.
Each question carries 15 *marks.*

1. (a) Define location-scale family of distributions. Give an example.

   (b) Define ancillary statistic with an example. State Basu's theorem.

   (c) Let X_1, X_2 be i.i.d. $B(1, \theta)$. Obtain an unbiased estimate of $\theta(1-\theta)$.

   (d) Distinguish between method of moments and method of minimum Chi-square.

   (e) Illustrate with an example that, MLE need not be unique.

(5 × 3 = 15 marks)

2. (a) Define one parameter exponential family of distributions. Show that $ET(X) = \frac{dA(\eta)}{d\eta}$ and

   $V(T(X)) = \frac{d^2A(\eta)}{d\eta^2}$ for this family.

   (b) Explain Fisher information and its properties.

(8 + 7 = 15 marks)

3. (a) Describe with examples (i) sufficient statistic ; (ii) minimal sufficiency ; (iii) complete family.

   (b) Suppose $(X_1, \ldots, X_n)$ is a random sample from $N(\theta, \theta)$, obtain sufficient statistic for θ. Check whether the sufficient statistic is complete.

   (c) State and prove Dynkin's theorem.

(6 + 6 + 3 = 15 marks)

4. (a) Suppose a random sample of size n is taken from Poisson (λ). Obtain any two unbiased estimators of λ. Which one is UMVUE ? Why ? Obtain the variance of UMVUE.

   (b) Illustrate any three properties of unbiased estimators.

(9 + 6 = 15 marks)

Turn over

5. (a) State and prove Rao-Blackwell and Lehmann–Scheffe theorems.

(b) Suppose $X_1, \ldots, X_n$ is a random sample from $P_{\theta_1, \theta_2}(x) = \frac{1}{(\theta_2 - \theta_1)}, \theta_1 \le x \le \theta_2$. Obtain UMVUE of θ_2.

(8 + 7 = 15 marks)

6. (a) State and prove CRK inequality. Obtain FCR inequality from it.

(b) Suppose $X_1, \ldots, X_n \sim N\left(\mu, \sigma_0^2\right)$, obtain FCRLB of variance of unbiased estimators of μ and μ^2.

(7 + 8 = 15 marks)

7. (a) Suppose $(X_1, \ldots, X_n) \sim \cup(a, b)$. Obtain the estimators of a and b by (i) method of moments and (ii) method of maximum likelihood estimation.

(b) State the asymptotic properties of MLE along with their assumptions.

(c) Prove or disprove : MLEs are functions of sufficient statistic.

(7 + 5 + 3 = 15 marks)

8. Write short notes on any *three* :

(a) Neyman-Fisher factorization theorem.

(b) Bounded completeness.

(c) Bhattacharya lower bounds.

(d) Method of scoring.

(3 × 5 = 15 marks)

(Pages : 2) **1581—II SS TA—MAY 2019**

SECOND SEMESTER M.A./M.Sc. DEGREE EXAMINATION, MAY/JUNE 2019

(CBCS)

Statistics

STCT 2.3—DEMOGRAPHY

Time : Three Hours Maximum : 75 Marks

Answer **Question No.1** *and any* **four** *from the remaining SEVEN questions.*
Each question carries 15 *marks.*

1. (a) Which are different sources of demographic data ?

   (b) Outline the difference between period and cohort fertility rates.

   (c) Write about the fertility measure using birth order statistics.

   (d) Define average crude death rate.

   (e) Define Quasi-stable population model.

   (5 × 3 = 15 marks)

2. (a) Explain coverage and content errors of demographic data.

   (b) State the assumptions of Chandrasekar-Deming (C-D) method of estimating population size N. State properties of $\widehat{N}$, an estimator of N obtained using C-D method. Also obtain its standard error.

   (c) Define Neonatal and perinatal mortality rates.

   (6 + 5 + 4 =15 marks)

3. (a) Describe indirect method of standardization of death rate.

   (b) Explain the use of Lexis diagram to compute adjusted measure of IMR.

   (c) Define Neonatal and Perinatal mortality rates.

   (6 + 5 + 4 =15 marks)

4. (a) Define : Fertility, Fecundity and TFR.

   (b) Explain the use of NRR in the study of population growth.

   (c) Write steps involved in estimating fertility levels using Brass P/F ratio.

   (3 + 5 + 7 =15 marks)

5. (a) State :

   (i) Uses of life tables.

   (ii) Assumptions made while constructing life tables.

Turn over

(b) Explain the columns of complete life table. Establish the inter relationship between m_x and q_x.

(6 + 9 = 15 marks)

6. (a) Describe Greville's method of constructing abridged life table.

(b) If d_i's are number of deaths in age interval (x_i, x_{i+1}), derive the joint distribution of d_i's.

(11 + 4 = 15 marks)

7. (a) Describe matrix method of population projection.

(b) Derive Lotka's stable population model. State its properties.

(7 + 8 = 15 marks)

8. Write short note on any three :

(a) Myer's Index.

(b) Shep-Perin stochastic reproductive process.

(c) Relationship between CDR and ASDR.

(d) Force of Mortality.

(e) Mean leangth of generation.

(3 × 5 = 15 marks)

(Pages : 3) **1583—II S STA—MAY 2019**

SECOND SEMESTER M.A./M.Sc. DEGREE EXAMINATION, MAY/JUNE 2019

(CBCS)

Statistics

OEC : STET 2.1—STATISTICAL METHODS

Time : Three Hours Maximum : 75 Marks

Question 1 *is* **compulsory.**

Answer any **four** *questions from remaining.*

Each question carries 15 *marks.*

1. (a) Explain any *two* sources of secondary data.

   (b) Define mode, state its merits and disadvantages.

   (c) Write a note on Geometric distribution.

   (d) Explain the use of scatter diagram in the study of correlation of two variables.

   (e) Define : Type I error, Type II error, Power of the test.

(5 × 3 = 15 marks)

2. (a) Explain any *two* methods of collections of primary data.

   (b) Prepare a frequency distribution table for the following data, using class interval of width 5 :

15	27	30	28	25	24	16	26	11	8
19	18	17	26	23	28	19	17	24	21
16	21	34	38	29	23	20	12	28	18
5	13	26	13	22	27	38	19		

   (c) Construct Ogives for the following data and find median value :

Class Interval	:	0–4	4–8	8–12	12–16	16–20	20–24
Frequency	:	7	13	16	22	24	9

(6 + 4 + 5 = 15 marks)

3. (a) Define arithmetic mean. Write its merits and demerits.

   (b) Find arithmetic mean for the following data :

C.I.	:	0–10	10–20	20–30	30–40	40–50	50–60
Frequency	:	4	7	11	18	6	3

Turn ove

(c) A boy travels from area A to area B at a speed of 10 Kms/hour. On his way back, he travels at a speed of 5 Kms/hour. Find the average speed.

(5 + 6 + 4 = 15 marks)

4. (a) Define various measures of dispersion.

(b) Find mean deviation from mean and standard deviation for the following data :

C.I.	:	5–10	10–15	15–20	20–25	25–30	30–35
Frequency	:	5	13	16	7	6	3

(7 + 8 = 15 marks)

5. (a) (i) Define Probability using axiomatic approach.

(ii) State addition and multiplication rules of probability.

(b) Consider the experiment of selecting one card at random from a deck of 52 playing cards. Find the probability that the card selected is :

(i) Either a spade or a face card.

(ii) Either a queen or a red color card.

(c) A physical therapist feels that scores on a certain manual dexterity test are approximately normally distributed with mean 10 and a standard deviation 2.5. If a randomly selected individual takes the test, what is the probability that he or she will make a score of (i) 15 or better (ii) atleast 16.

(4 + 6 + 5 = 15 marks)

6. (a) State the properties of correlation co-efficient.

(b) What are the assumptions underlying simple linear regression analysis.

(c) The following data are collected to study the heart rate measurements (x) and anxiety measurements (y) :

x	:	50	55	60	70	75	80	85	90	95
y	:	48	41	45	41	42	36	30	32	34

Obtain the regression equation and find anxiety measurement when heart rate measures 100.

(3 + 3 + 9 = 15 marks)

7. (a) Explain z-test for assumed mean.

(b) For the following data set, test the hypothesis of population mean is not equal to 12 at 5 % level of significance :

14.0, 14.2, 13.2, 14.5, 11.2, 14.0, 14.1, 12.2, 11.1, 13.7, 13.2, 16.0, 12.8, 14.4, 12.9

(c) Explain the use of χ^2-test for 2×2 contingency table.

(3 + 7 + 5 = 15 marks)

8. Write short note on any *three* :

(a) Kurtosis.

(b) Normal Distribution.

(c) Relation between regression and correlation.

(d) Non-parametric tests.

($3 \times 5 = 15$ marks)