8995—IV S STA—Sep. 2021

FOURTH SEMESTER M.A./M.Sc. (CBCS) DEGREE EXAMINATION, SEPTEMBER 2021

Statistics

STCT 4.1—MULTIVARIATE ANALYSIS

Time : Three Hours | Maximum : 75 Marks

Answer any **five** *questions, including* **Question 1** *which is* **compulsory**.
Each question carries 15 *marks.*

1. (a) Suppose $\underline{Y} = A\underline{X} + \underline{b}$ and $\text{Cov}(\underline{X}) = \Sigma$, then show that $\text{Cov}(\underline{Y}) = A\Sigma A'$.

   (b) State and prove any one application of Hotelling's T^2.

   (c) Write the assumptions of one-way MANOVA.

   (d) What is discrimination and classification ?

   (e) Explain the three types of clustering procedures.

(5 × 3 = 15 marks)

2. (a) Let $\underline{X}_1, \underline{X}_2, \dots, \underline{X}_n \sim N(\underline{\mu}, \Sigma)$, then show that $\bar{\underline{X}}$ is an unbiased estimator of the population mean vector $\underline{\mu}$ and the sample variance-covariance matrix with divisor $(n-1)$ is an unbiased estimator of Σ.

   (b) State and prove Cramer-Wald Theorem.

(10 + 5 = 15 marks)

3. (a) Let $\underline{X}_1, \underline{X}_2, \dots, \underline{X}_n \sim N(\underline{\mu}, \Sigma)$, then find the maximum likelihood estimator of $\underline{\mu}$ and Σ.

   (b) Prove that the distribution of the random vector $\underline{X}_{p\times 1}$ is known iff the distribution of $\underline{\alpha' X}$ is known $\forall \underline{\alpha} \in R^p$.

(10 + 5 = 15 marks)

4. (a) Let $\underline{u} \sim N(\underline{\mu}, \Sigma)$, where Σ is not specified and $D \sim N(\Sigma, f)$. Suppose $\underline{u}$ and D are independent. Define $T^2 = f\underline{u}' D^{-1}\underline{u}$. Then show that $\dfrac{T^2}{1}\dfrac{f-p+1}{p}$ is defined as the non-central $F_{p,f-p+1}$.

   (b) Let $\underline{X} \sim N(\underline{\mu}, \Sigma)$, where Σ is a positive definite matrix then show that $(\underline{X}-\underline{\mu})' \Sigma^{-1} (\underline{X}-\underline{\mu}) \sim \chi^2_p$.

(11 + 4 = 15 marks)

5. (a) Define Wishart distribution.

   (b) Suppose $A_1 \sim W_m(n_1, \Sigma)$ and $A_2 \sim W_m(n_2, \Sigma)$ and are independent, then show that $A_1 + A_2 \sim W_m(n_1 + n_2, \Sigma)$.

   (c) Suppose $A \sim W_m(n, \Sigma)$ and let C be a $q \times m$ non-random matrix then show that $CAC' \sim W_q(n, C\Sigma C')$.

(3 + 6 + 6 = 15 marks)

Turn over

6. (a) In multivariate multiple regression case, obtain MLE of β and Σ.

   (b) Explain any two properties of MLE.

(10 + 5 = 15 marks)

7. (a) Obtain the Fisher Linear Discriminant Function.

   (b) Explain the relationship between TPM minimizing rule and Baye's classification rule.

(10 + 5 = 15 marks)

8. Write any *three* :

   (a) Define Wishart distribution. Discuss Wishart-Bartlet lemma.

   (b) Write the two-way MANOVA with interaction table.

   (c) Explain the ECM strategy for classification rule.

   (d) Explain multi-dimensional scaling and mention its types.

(3 × 5 = 15 marks)

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FOURTH SEMESTER M.A./M.Sc. DEGREE (C.B.C.S.) EXAMINATION
SEPTEMBER 2021

Statistics

Paper STCT 4.2—LINEAR MODELS

Time : Three Hours | Maximum : 75 Marks

Question 1 *is* compulsory.

Answer any **four** *questions from the remaining seven questions.*
Each question carries 15 *marks.*

1. Answer *all* the following questions :

   a) Define a multi-variate linear regression model and state its assumptions.

   b) Define best linear unbiased estimator (BLUE) and explain how to choose BLUE.

   c) Explain linear estimation in case of restrictions.

   d) Define incomplete block design and distinguish it from the complete block designs.

   e) Show that BIBD is a balanced design.

(5 × 3 = 15 marks)

2. a) State and prove Gauss-Markov (G-M) theorem.

   b) Under Gauss-Markov Setup $\left(y, X\beta, \sigma^2 I\right)$ for the linear model $y = X\beta + \in$, the least squares estimate of β is $\hat{\beta} = \left(X' X\right)^{-1} X' y$. Then show that the unbiased estimator of σ^2 based on least square estimate is $\hat{\sigma}^2 = \frac{1}{n-p}\left(y - X\hat{\beta}\right)'\left(y - X\hat{\beta}\right)$.

   c) Consider the model $E\left(y_1\right) = \beta_1 + \beta_2, E\left(y_2\right) = \beta_1 - \beta_2$ and $E\left(y_3\right) = \beta_1 + 2\beta_2$ with the usual assumptions. Examine whether the linear parametric functions β_i is estimable ? If so obtain its BLUE.

(6 + 5 + 4 = 15 marks)

Turn over

3. a) Derive variance and covariance of least squares estimates.

 b) Explain linear estimation in the restriction of parameters. Derive the least squares estimate of β under G-M setup.

 c) Define quadratic forms. State any *three* applications of under G-M setup.

(5 + 7 + 3 = 15 marks)

4. a) Under G-M setup $\left(y, X\beta, \sigma^2 I\right)$, derive the likelihood ratio test (LRT) for testing $H_0 : K'\beta = m$ *v/s* $H_1 : K'\beta \neq m$. Where K is a $p \times q$ matrix with rank (K) = q and m is $q \times 1$.

 b) Explain the application of Gauss-Markov theorem in two-way classification

(7 + 8 = 15 marks)

5. a) Define estimation space and error space. State their properties.

 b) Explain test of hypothesis for a single contrast and derive confidence intervals for the contrast.

 c) Explain Tukey's method for pairwise comparisons and derive Tukey's simultaneous confidence intervals. Compare Tukey's simultaneous confidence intervals with Scheffe's Tukey's simultaneous confidence intervals. (3 + 5 + 7 = 15 marks)

6. a) What is missing plot technique ? Obtain the estimate of single missing observation in randomized complete block design (RCBD). Explain the exact analysis of RCBD and obtain the bias in this case.

 b) Define analysis of covariance (ANCOVA). Explain its in case of completely randomized design with single covariate. (7 + 8 = 15 marks)

7. a) Define a balanced incomplete block design (BIBD) and state and prove its necessary conditions.

 b) Explain the intra-block analysis of BIBD.

 c) Define symmetric BIBD. Show that in a symmetric BIBD, the number of treatments common between any blocks is λ. (5 + 6 + 4 = 15 marks)

8. a) Show that the reduced treatment totals are orthogonal to the block totals in the sense that, $\text{Cov}\left(Q_i, B_j\right) = 0$ for all $i \neq j$.

 b) Show that intra-block and recovery of inter-block estimators are independent.

 c) Define random effects models. Stating all the assumptions involved, describe one-way random effects model. Give an example where in this model can be used.

(4 + 4 + 7 = 15 marks)

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FOURTH SEMESTER M.A./M.Sc. DEGREE (C.B.C.S.) EXAMINATION, SEPTEMBER 2021

Statistics

Paper STCT 4.3—STATISTICAL QUALITY CONTROL AND RELIABILITY THEORY

Time : Three Hours

Maximum : 75 Marks

Question 1 *is* compulsory.

Answer any **four** *questions from the remaining questions.*

Each sub-question carries 15 *marks.*

1. a) Distinguish between process control and product control.

   b) Define : OC function, Average run length and Process capability ratio.

   c) Explain the operations of chain sampling plan.

   d) Define IFR and IFRA classes of life distributions for both continuous and discrete case.

   e) Describe Stress-Strength models.

(15 marks)

2. a) Explain in brief the steps involved in the construction of $\bar{X}$ and S charts.

   b) Name the different criterion set by Nelson for a process to be out of control.

   c) Describe the importance and construction of group control charts.

(6 + 4 + 5 = 15 marks)

3. a) Describe a moving average control chart and state its advantages over a Shewhart's control chart.

   b) Explain in brief the construction of χ^2 control charts. State its disadvantages.

   c) When are sloping control charts necessary. Write down the sloping control limits.

(5 + 7 + 3 = 15 marks)

4. a) What is an acceptance sampling plan ? State its uses.

   b) Explain the single sampling plan and obtain its OC function.

   c) Describe CSP1 and CSP2 plans.

(3 + 5 + 7 = 15 marks)

5. a) Show that Binomial distribution has IFR.

   b) Prove that IFR => IFRA => NBU => NBUE.

(6 + 9 = 15 marks)

6. a) Describe the following :

      i) Stress-Strength model.

      ii) Proportion hazard model.

   b) What is a parallel system ? Obtain there liability of a parallel system with k components. What happens to the reliability of a k component parallel system if all the components are independent ?

   c) Compare age and block replacement policies.

(6 + 6 + 3 = 15 marks)

7. a) Define renewal random variable, renewal function and obtain the distribution of the renewal random variable.

   b) Show that the renewal function $M(t) = E[N(t)]$ satisfies $M(t) = F(t) + \int_0^{\infty} M(t-x)\, dF(x)$.

   c) Test the hypothesis of mean life of the components equal to a specified value against the alternative that mean life is less than the specified value, when the number of survivals during $(0, t)$ is observed in case of exponential distribution.

(4 + 4 + 7 = 15 marks)

8. Write short notes on any *three* of the following :

   a) EWMA control charts.

   b) CUSUM control chart.

   c) Censored samples.

   d) Shock models.

(3 × 5 = 15 marks)