8537 – IIS STA – S – 21

SECOND SEMESTER M.A./MSc. (CBCS) DEGREE EXAMINATION, SEPTEMBER 2021

STATISTICS

Paper : STCT 2.1 – (Probability Distributions)

Time : 3 Hours] [Max. Marks : 75

Answer **Q1** *and any* **four** *from the remaining* **Q2** *to* **Q8**.
Each question carries equal marks.

1. a) Prove that the conditional distribution of Binomial distribution is a Hypergeometric distribution. 5×3

   b) Obtain the mode of Logarithmic series distribution.

   c) Show that the mean of Cauchy distribution does not exist.

   d) Define central F-distribution. State its applications.

   e) Describe compound distribution with an example.

2. a) If $x \sim B(n, p)$, obtain mean absolute deviation of X. $7 + 4 + 4$

   b) State and prove memoryless property of Geometric distribution.

   c) Obtain mgf of Multinomial distribution.

3. a) Find mean and variance of zero truncated Poisson distribution. $7 + 8$

   b) For lognormal distribution, show that the Mode < Median < Mean.

4. a) If x_1 and x_2 are continuous uniform random variables over (0, 1), find the distribution of $x_1 + x_2$ and $x_1 - x_2$. $7 + 8$

   b) Suppose U and V are independent and identically distributed as $f(x) = \frac{\sqrt{2}}{x(1 + x^4)}$, $-\infty < x < \infty$. Find the distribution of $\frac{U}{V}$.

5. a) Explain the three forms of Gumbel's bivariate exponential distribution. $(5 + 5 + 5)$

   b) State and prove additive property of Gamma distribution.

   c) Suppose X and Y are iid standard exponential random variables, then show that $U = X - Y$ has double exponential distribution.

[P.T.O.

6. a) Derive the density function of single order statistics. (5 + 4 + 6)

b) Let $X_{1,n} \le X_{2,n} \le .... \le X_{n,n}$ be order statistics for a sample of size n from an absolutely continuous distribution F(x). Then show that the conditional distribution of $X_{s,n}$ given $X_{r,n} = X_r$ for $r < s$, is same as the distribution of $(s - r)^{th}$ order statistics from a sample of size $(n - r)$ from a distribution F(x).

c) Obtain the density function of median in a sample of size $n = 2m$ from exponential distribution with density function $f(x) = e^{-x}$, $x > 0$.

7. a) Derive the density function of non-central t-distribution.

b) Let $X_1, X_2, ..., X_n$ be the independent normal random variables with mean μ and variance σ^2, then show that $\overline{X}$ and $(X_1 - \overline{X}, X_2 - \overline{X}, ...X_n - \overline{X})$ are independent. 8 + 7

8. a) Find the density function of sample range for a sample of size n from uniform distribution with density $f(x) = 1 \,;\, 0 < x < 1$. 5 + 5 + 5

b) If X_1, X_2, X_3, X_4 are independent N(0, 1), find the distribution of $(X_1 X_2 + X_3 X_4)$.

c) Describe logistic distribution as growth function.

8538 – IIS STA – S – 21

SECOND SEMESTER M.A./M.Sc. (CBCS) DEGREE EXAMINATION, SEPTEMBER 2021

STATISTICS

Paper : STCT 2.2 – (Theory of Point Estimation)

Time : 3 Hours] [Max. Marks : 75

Answer any **five** *questions, including question no.* **1.**

Question no. **1** *is* **compulsory.**

Each question carries **15** *marks.*

1. a) Define one parameter exponential family of distributions. Give an example. $5 \times 3 = 15$

   b) Define Fisher information. Obtain Fisher information contained in a single B(n, p) random variable (rv).

   c) Explain the concepts of sufficiency and minimal sufficiency.

   d) With an example, show that "unbiased estimator may not always exist."

   e) Define CRK inequality and discuss about its application.

2. a) Suppose X_1 and X_2 are r vs from $P(\lambda)$. Show that $6 + 9$

   (i) $X_1 + X_2$ is sufficient statistic and ii) $X_1 + 2X_2$ is not sufficient statistic for λ.

   b) State and prove Neyman Fisher factorization theorem for discrete case. Discuss about its applications and limitation.

3. a) Distinguish between complete and bounded complete family of distributions. Give one example from each of these families. $5 + 4 + 6$

   b) Discuss any two properties of Fisher information.

   c) Define ancilliary statistic. State and prove Basu's theorem.

4. a) If $X_1, \ldots\ldots\ldots, X_n$ are rvs from $U(\theta - \frac{1}{2}, \theta + \frac{1}{2})$, obtain minimal sufficient statistic for θ.

   b) Suppose $X_1, \ldots\ldots, X_n$ are rvs from $N(\theta, \theta^2)$. Obtain minimal sufficient statistics for the family and show that this statistic is not complete.

   c) Show that complete sufficient statistic is always minimal sufficient. $4 + 7 + 4$

5. a) Define unbiased estimator. Suppose $X_1, \ldots\ldots, X_n$ is a random sample from $U(0, \theta)$, obtain any three unbiased estimators of θ. Also, obtain variances of these estimators. $10 + 5$

   b) Distinguish between LMVUE and UMVUE. State Rao-Blackwell and Lehmann-Scheffe theorems.

[P.T.O.

6. a) State and prove Cramer-Rao inequality.

   b) Suppose $X_1, \dots, X_n$ is a random sample from $N(\theta, 1)$, check whether the variances of UMVUE of θ and θ^2 attain Cramer-Rao lower bound. 7 + 8

7. a) Outline the method of moment estimation. Illustrate with an example that "moment estimators need not be unique". 7 + 8

   b) Explain the method of maximum likelihood estimation. Suppose $X_1, \dots, X_n$ is a rs from $f_{\alpha,\beta}(x) = \beta e^{-\beta(x-\alpha)}$, $x > \alpha$, obtain mles of α and β.

8. Write short notes on any **three**: $3 \times 5 = 15$

   a) Group family of distributions

   b) Bhattacharya lower bounds.

   c) Asymptotic properties of mle.

   d) Chi-square and modified Chi-square methods of estimation.

8539 – IIS STA – S – 21

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STATISTICS

Paper : STCT 2.3 – (Demography)

Time : 3 Hours] [Max. Marks : 75

Answer Q.No. **1** *and any* **four** *from the remaining* **seven** *questions.*

1. a) Explain age heaping and tendentious bias with examples. 5×3

   b) Distinguish between period and cohort fertility measures. Explain the different sources of fertility data.

   c) Define complete and abridged life tables. Explain the assumptions in the construction of life table.

   d) Define mean length of generation and derive its expression for stable population using second approximation to intrinsic growth rate.

   e) Define population estimation and projection. Explain exponential method of estimating population growth.

2. a) What are the different sources of demographic data and explain vital statistics registration system. 3 + 8 + 4

   b) Explain Chandrasekharan and Deming method of ascertaining the completeness of vital statistics registration.

   c) Explain United Nation's index.

3. a) Define Crude Death Rate (CDR) and Age-Specific Death Rate (ASDR). Give their limitations and also obtain the relationship between them. 4 + 6 + 5

   b) Define i. Infant Mortality Rate (IMR) ii. Neo-Natal Mortality Rate (NMR) and iii. Post-NMR. Explain the use of Lexis chart for estimating IMR.

   c) Explain different methods of standardization of death rates.

4. a) Define i. Crude Birth Rate (CBR) ii. General Fertility Rate (GFR) and iii. Total Fertility Rate (TFR). Establish the relationship between these rates. 5 + 4 + 6

   b) Explain child-women ratio. Give its importance and limitations.

   c) Explain Brass P/F ratio method of estimating total fertility rate.

[P.T.O.

5. a) Explain different columns of complete life table and establish their interrelationships. 4 + 4 + 7

b) Define force of mortality. With usual notations establish the relation between ${}_nq_x$ and μ_x.

c) Stating the assumptions, explain the Greville's method of construction of abridged life table.

6. a) Explain Shep's and Perrin model of human reproductive process. 7 + 8

b) Explain the component method of estimating population.

7. a) Distinguish between stationary and stable population models. Stating the underlying assumptions, derive Lotka's stable population model. 8 + 3 + 4

b) With usual notation, for a stable population show that $r = \frac{\log_e R}{T}$.

c) Explain the momentum of population growth.

8. Write short notes on any **three** of the following: 3 × 5

a) Whipple's index of estimating the error in age reporting

b) Relationship between ${}_nq_x$ and ${}_nm_x$

c) Reproduction and replacement rates

d) Quasi stable population.