8530-IISSTA-M-18

M.A./M.Sc. DEGREE EXAMINATION MAY 2018.

Second Semester

(CBCS)

STATISTICS

Paper STCT 2.1 – PROBABILITY DISTRIBUTIONS

Time : Three hours

Maximum : 75 marks

Question **No. 1** is Compulsory.

Answer any **FOUR** from the remaining questions.

Each question carries **15** marks.

1. (a) The joint distribution of X and Y is given by $f(x, y) = 4\, xy e^{-(x^2+y^2)}$ $x \geq 0$, $y \geq 0$. Test whether X and Y are independent.

   (b) If X and Y are two independent geometric random variables, show that the conditional distribution of $X / X + Y = n$ is uniform.

   (c) Derive interquartile range of Cauchy distribution.

   (d) Obtain mode of Central Chi squared distribution.

   (e) What is compound distribution? Give an example. (5 × 3 = 15)

2. (a) Let the random variable X have the geometric distribution with parameter 'p', i.e. $P(X = x) = pq^{x-1}$, $x = 1, 2, \ldots$.

   (i) Show that $P(X > m) = q^m$, where m is a positive integer.

   (ii) Prove that $P(X > m + n \mid X > m) = P(X > n)$ where m and n are positive integer.

   (b) Obtain mode of lognormal distribution. (8 + 7)

3. (a) Establish mean absolute deviation about mean of binomial distribution.

   (b) Suppose X and Y are iid standard exponential random variables, obtain distribution of $X - Y$. (8 + 7)

4. (a) Let $X \sim B(a, b)$, i.e. $f_X(x) = \dfrac{x^{a-1}(1-x)^{b-1}}{B(a, b)}$, $0 \leq x \leq 1$, $a, b > 0$.

   (i) Find $E(X)$ and $V(X)$

   (ii) Show that $1 - X$ has Beta distribution.

   (b) Define logistic distribution. Obtain its mgf. (8 + 7)

5. (a) Obtain moment generating function of bivariate normal distribution.

(b) Disçuss the Three different forms of Gumbel's bivariate exponential distribution. Derive its conditional expectation of Y given $X = x$ and X given $Y = y$. (7 + 8)

6. (a) Let $Y_1 < Y_2 < \ldots < Y_n$ be the order statistics of a random sample from a distribution with pdf.

$$f(x) = \begin{cases} e^{-x}, & x > 0 \\ 0, & 0.w \end{cases}$$

Let $Z_n = Y_n - \log n$. Find the cdf of Z_n.

(b) Suppose that $X_1, X_2, \ldots, X_n$ are iid continuous random variables with cdf $F(x)$. Let $X_{(1)} \le X_{(2)} \le \ldots \le X_{(n)}$ be the order statistics. Find the joint distribution of $(X_{(1)})$ and $(X_{(n)})$. (7 + 8)

7. (a) Derive central F-distribution. Find its mean and variance.

(b) If X_i $(i = 1, 2, \ldots, K)$ are independent normal variate with mean μ_i $(i = 1, 2, \ldots, K)$ and variance σ_i^2 $(i = 1, 2, \ldots, K)$. Then obtain the probability distribution of $a_1X_1 + a_2X_2 + \cdots + a_kX_k$. (10 + 5)

8. (a) Obtain mean and variance of truncated Poisson distribution.

(b) If $X_1, X_2 \sim N(\mu_i, 1)$, $i = 1, 2$. Find the distribution of $\left(\frac{X_1 + X_2}{\sqrt{2}}\right)^2$ and $\left(\frac{X_1 - X_2}{\sqrt{2}}\right)^2$. (8 + 7)

8531-IISSTA-M-18

M.A./M.Sc. DEGREE EXAMINATION MAY 2018.

Second Semester

(CBCS)

STATISTICS

Paper STCT 2.2 – THEORY OF POINT ESTIMATION

Time : Three hours

Maximum : 75 marks

Answer any **FIVE** questions, including Question **No. 1** is Compulsory.

Each question carries **15** marks.

1. (a) Check whether Gamma (α, β), α known, $\beta > 0$ distribution belongs to one parameter exponential family of distributions.

(b) Explain Fisher Information (FI). Prove that FI contained in n random observations is as same as n times FI contained in a single random observation.

(c) Define minimal sufficient statistic. Suppose a random sample (rs) of size n is taken from Bernoulli (1, p), obtain minimal sufficient statistic for p.

(d) Explain the methods of obtaining UMVUE of a parameter.

(e) Distinguish between methods of minimum chi-square and minimum modified Chi-square. (5 × 3 = 15)

2. (a) Define S-parameter Koopman-Darmois family of distributions. Give an example.

(b) Obtain FI contained in a r.v. of one-parameter exponential family of distributions.

(c) Show that for location family of distributions, FI is independent of parameter. Hence obtain FI in a single r.v. that belongs to logistic distribution with location parameter θ. (4 + 4 + 7)

3. (a) Explain the concept of sufficiency. Explain the method of obtaining a sufficient statistic. Hence obtain sufficient statistics in the following cases :

(i) $f_{\alpha,\beta}(x) = \beta\, e^{-\beta(x-\alpha)}, \quad x > \alpha$

(ii) $f_{\alpha,\beta}(x) = \dfrac{1}{\beta - \alpha}, \quad \alpha < x < \beta.$

(b) Suppose X_1, X_2 are r vs from $P(\lambda)$, check whether the following statistics are sufficient or not.

(i) $X_1 + X_2$ and (ii) $2X_1 + X_2$. (9 + 6)

4. (a) Define and distinguish between bounded complete and complete family of distributions. Give one example from each of these families.

   (b) Define ancillary statistic with an example.

   (c) State and prove Basu's theorem. Illustrate its application. (6 + 3 + 6)

5. (a) Define an unbiased estimator. Suppose a r.s. of size n is taken from $N(\mu, \sigma^2)$, μ unknown. Obtain an unbiased estimator of σ.

   (b) Define UMVUE. Suppose a rs of size n is drawn from $N(0, \sigma^2)$, obtain UMVUE of σ^2 and $\frac{1}{\sigma^2}$. (8 + 7)

6. (a) State and prove characterization theorem of UMVUE.

   (b) State Cramer-Rao inequality. Suppose $X_1, \ldots X_n$ be a random sample of size n from Poisson (λ), obtain an unbiased estimator of λ^2 and check whether it's variance attains Cramer-Rao lower bound. (7 + 8)

7. (a) Explain : (i) likelihood function (ii) method of moment estimation and (iii) maximum likelihood estimator (MLE) giving one example each.

   (b) Establish the relationship between sufficiency and MLE.

   (c) Illustrate that the moment estimators need not be unique. (6 + 5 + 4)

8. (a) Suppose $X_1, \ldots, X_n$ is a r.s. from following distributions. Obtain MLE in each case.

   (i) $f_{\theta,\lambda}(x) = \frac{1}{\lambda} e^{-\frac{(x-\theta)}{\lambda}}$, $x \geq \theta,\ \lambda > 0$ and

   (ii) $f_\theta(x) = \theta x^{\theta - 1}$, $0 < x < 1$, $\theta > 0$

   (b) With an example, show that MLE need not be unbiased.

   (c) Illustrate the method of scoring to obtain MLE. (7 + 3 + 5)

8532-IISSTA-M-18

M.A./M.Sc. DEGREE EXAMINATION MAY 2018.

Second Semester

(CBCS)

STATISTICS

Paper STCT 2.3 – DEMOGRAPHY (OPTIONAL)

Time : Three hours Maximum : 75 marks

Question **No. 1** is Compulsory.

Answer any **FOUR** from the remaining questions.

Each question carries **15** marks.

1. (a) Explain Whipple's index of identifying digit preference in age reporting.

   (b) Define crude death rate (cdr) and age-specific death rate (asdr). Explain their limitations.

   (c) Define cohort and period life table. State the assumptions in the construction of life table.

   (d) With usual notation for a stable population, show that $r = \frac{\log_e R}{r}$.

   (e) Define population estimation and projection. Explain Gumbel's method of estimating population growth. (5 × 3)

2. (a) Explain the different sources of demographic data.

   (b) Explain Chandrasekharan and Deming method of ascertaining the completeness of vital statistics registration.

   (c) Distinguish between coverage and content errors. (4 + 8 + 3)

3. (a) Establish the relationship between cdr and asdr.

   (b) Define : (i) Infant mortality rate (IMR) and (ii) Neo-natal mortality rate (NMR). Explain the different methods of estimating IMR.

   (c) Explain standardized death rates. (3 + 7 + 5)

4. (a) Explain child-women ratio and give its limitations.

   (b) Define : (i) Crude birth rate (cbr) (ii) General fertility rate (gfr) and (iii) Total fertility rate (tfr). Establish the relationship between these rates.

   (c) Define reproduction and replacement rates. Explain net reproduction rate. (4 + 6 + 5)

5. (a) Define force of mortality. With usual notations establish the relation between (i) ${}_nq_x$ and ${}_nm_x$ and (ii) ${}_nq_x$ and μ_x.

   (b) Stating the assumptions, explain the Greville's method of construction of abridged life table. (6 + 9)

6. (a) Explain Brass P/F ratio method of estimating total fertility rate.

   (b) Explain the component method of estimating population. (6 + 9)

7. (a) Define stationary and stable population models. Stating the underlying assumptions of Lotka's stable population model, derive the maternity function.

   (b) Derive an expression for mean length of generation using second approximation to intrinsic growth rate.

   (c) Define intrinsic birth and death rate and give their relationship with age structure. (7 + 5 + 3)

8. Write short note on any **THREE** of the following :

   (a) Myrse's index of estimating the error in age reporting

   (b) Reed-Merrel method of construction abridged life table

   (c) Momentum of population growth

   (d) Quasi stable population. (3 × 5)

8534-IISSTA-OE-M-18

M.A./M.Sc. DEGREE EXAMINATION MAY 2018.

Second Semester

(CBCS)

STATISTICS

Paper STET 2.1 – STATISTICAL METHODS

Time : Three hours　　　　Maximum : 75 marks

Question No. 1 is Compulsory.

Answer any **FOUR** from the remaining questions.

Each question carries **15** marks.

1. (a) Write various sources of secondary data.

   (b) Define kurtosis, name its types.

   (c) Define :

      (i) Exhaustive events

      (ii) Mutually exclusive events

      (iii) Independent events

   (d) Explain the use of scatter diagram in the study of correlation

   (e) Define :

      (i) Type I error

      (ii) Size of the test

      (iii) Power of the test

2. (a) Write the rules for tabulation of data.

   (b) Following is the monthly expenses of 3 families. Construct component bar diagram.

Item	Monthly Expenses		
	A	B	C
Food	1000	1400	2000
HR	500	750	1000
Fuel	200	300	300
Others	800	1550	3000

(c) Construct Pie-diagram for the following data on expenses incurred by a company on different aspects.

Item	Yearly expenses in Rs.
Raw materials	18,00,000
Rent & Electricity	9,00,000
Salary	4,50,000
Transportation	4,50,000
Others	19,00,000

(5 + 5 + 5)

3. (a) What property should a good measure of central tendency possess?

(b) Define Median and Mode along with their merits and limitations.

(c) Find the arithmetic mean for the following data : (4 + 4 + 7)

C.I	Frequency
10–20	23
20–30	44
30–40	35
40–50	12
50–60	9
60–70	3
70–80	2

4. (a) Define :

(i) Co-efficient of quartile deviation

(ii) Co-efficient of mean deviation

(iii) Co-efficient of variation

(b) The marks scored by two students in 8 tests are as follows. Find who is better scorer and who is consistent scorer? (6 + 9)

Std 1 :	31	48	25	55	38	43	50	36
Std 2 :	51	36	42	83	37	18	42	20

5. (a) Three coins are tossed simultaneously. Find the probability of getting at least two heads.

(b) Define Poisson distribution. State its properties.

(c) The wages of 3000 workers are normally distributed with mean of Rs. 100 and standard deviation of Rs. 10. Estimate the number of workers whose wages will be (i) between Rs. 80 and Rs. 110 (ii) more than Rs. 120. (5 + 4 + 6)

6. (a) State the differences between correlation and regression.

   (b) Obtain the regression line of Y on X from the information :

	X	Y	
Mean	45	30	$r_{xy} = 0.55$
SD	10	6	

   (c) Calculate Karl Pearson's correlation co-efficient : (5 + 3 + 7)

X :	12	16	20	23	25	28	31
Y :	23	28	33	42	47	50	55

7. (a) A sample of 100 students is selected from a population. The mean of selected students is 48 kg with a standard deviation of 5 kg. Test whether the population mean weight is 50 kg.

   (b) Explain the use of t-test.

   (c) Following table gives the information above the numbers of persons inoculated, not inoculated, infected and not infected. Test whether inoculation is effective in controlling the infection. (5 + 3 + 7)

	Infected	Not infected
Inoculated	55	105
Not-inoculated	140	100

8. Write short note on any **THREE** :

   (a) Skewness

   (b) Non-parametric tools

   (c) Binomial-distribution

   (d) Method of least squares (3 × 5)