(Pages : 2) **1584—II S STA (PR)—MAY 2019**

SECOND SEMESTER M.A/M.Sc. DEGREE EXAMINATION, MAY/JUNE 2019

(CBCS)

Statistics

**Paper STCP 2.1—BASED ON STCT 2.1 (PROBABILITY DISTRIBUTION)**

Time : Two Hours

Maximum : 30 Marks

*Answer any* **three** *questions.*
*Each question carries* 10 *marks.*

1. (a) Out of 20 packages to be dispatched by a mail-room Clerk eight are to be sent by air mail and the rest by surface mail. The packages got mixed up thoroughly. Five of the packages are selected randomly. The distribution of packages marked for air mail getting into the chosen five packages, observed over period of 100 days is :

| X | : | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|
| $f$ | : | 7 | 22 | 45 | 20 | 5 | 1 |

Fit a suitable distribution for the above data and test the goodness of fit.

(b) Suppose that $P(x, y)$, the joint probability mass function of X and Y is

$P(1,1) = 0.5, P(1,2) = 0.1, P(2,1) = 0.1, P(2,2) = 0.3$. Calculate pmf of X given that $Y = 1$.

(8 + 2 = 10 marks)

2. (a) Suppose I invite 11 people to a party, who all arrive independently of one another. The arrival times of each person are i.i.d. variables, each with density $f(x) = 2x/9$ for $x \in [0, 3]$ ($x$ is the time in hours from 8:00 pm). Find :

(i) The density for the arrival time of the 8th person.

(ii) The probability that the 8th person to arrive gets there between 9:00 and 10:00.

(b) The annual flow, in cubic meters per second (m 3/s), at the Weldon River at Mill Grove Missouri, for the period 1930 to 1960 are averages as given........Fit the lognormal distribution to this data using the Method of Moments. What is the probability that the annual river flow is in the range 15 to 2 m 3/s.? or, $P(2 \leq X \leq 15) = ?$

$(m_x = 7.3081 \text{ and } s_x = 5.5296)$.

(6 + 4 = 10 marks)

**Turn over**

3. (a) The annual runoff in the Cave Creek watershed near Fort Spring, Kentucky, USA are given as follows in millimeters over an 18-year period :

337, 84, 394, 361, 538, 196, 448, 582, 480, 326, 294, 385, 264, 458, 413, 299, 455, 291

Assuming independence and a gamma distribution for the annual runoff, Determine the probability that the runoff will be greater than 100 mm in a given year.

(b) Observations on average annual discharge (represented as random variable X) on a stream are as given below :

| Obs. No. | Discharge ($m^3/s$ | Obs. No. | Discharge ($m^3/s$ | Obs. No. | Discharge ($m^3/s$ | Obs. No. | Discharge ($m^3/s$ |
|---|---|---|---|---|---|---|---|
| 1 | 120 | 7 | 51 | 13 | 135 | 19 | 150 |
| 2 | 105 | 8 | 41 | 14 | 280 | 20 | 106 |
| 3 | 95 | 9 | 30 | 15 | 420 | 21 | 73 |
| 4 | 81 | 10 | 28 | 16 | 470 | 22 | 65 |
| 5 | 73 | 11 | 30 | 17 | 415 | 23 | 66 |
| 6 | 62 | 12 | 52 | 18 | 260 | 24 | 110 |

The volume of runoff in an adjacent stream is functionally represented as $Y = 3\ln X + 5$. If the discharge X follows lognormal distribution, find $P(Y \geq 10)$.

(4 + 6 = 10 marks)

4. (a) Fit a zero truncated binomial distribution to the following data :

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $x$ | : | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| $f$ | : | 5 | 16 | 32 | 30 | 23 | 7 | 6 |

Test the goodness of fit.

(b) Write a program to generate uniform (0, 1) random numbers and generate 100 random numbers.

(7 + 3 = 10 marks)

[3 × 10 = 30 marks]

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## STCP 2.2—PRACTICALS (BASED ON STCT 2.2)

Time : Two Hours

Maximum : 30 Marks

*Answer any* **three** *of the following questions.*
*Each question carries* 10 *marks.*

1. (i) Show that the following family of distributions is one parameter exponential family. Hence find its mean :

$$P_\pi(x) = \frac{\binom{n}{x}\pi^x(1-\pi)^{n-x}}{1-(1-\pi)^n}, \quad x = 1, 2, \ldots\ldots, n.$$

(ii) Find Fisher information contained in $X_i, i = 1, \ldots, 10$ about $\theta$ if $X_i$ Logistic $(\theta, 1)$.

(iii) 8.63, 23.5, 16.43, 19.02 and 12.61 are random samples from Poisson $(\lambda)$. Find the value of the sufficient statistic.

(3 + 4 + 3 = 10 marks)

2. (i) 16.59, – 13.46, 12.98, 15.63, 17.21, 13.12 are from N $(\theta, \theta)$. Find the value of minimal sufficient statistic.

(ii) Find and compute a complete sufficient statistic for $\theta$ when 0.39, 0.89, 0.96, 0.58, 0.43, 0.12, 0.58, 0.19, 0.59, 0.48, 0.38, 0.27, 0.19, 0.21 and 0.16 is a random sample from

$$f_\theta(x) = \theta x^{\theta-1}, \quad 0 < x < 1, \theta > 0.$$

(5 + 5 = 10 marks)

3. (i) A random sample of 12 observations from U $(0, \theta)$, is given below :

0.7, 1.0, 1.3, 1.4, 1.6, 1.5, 0.8, 0.5, 1.3, 1.7, 0.4, 0.2. Obtain two unbiased estimates of $\theta$ and obtain their estimates of the variance. Give your comments.

Turn over

(ii) A random variable X has exponential distribution with mean $\frac{1}{\theta}$. Obtain UMVUE of $P(X < 1)$ based on a sample of size 2 with $X_1 = 1/2, X_2 = 3$.

(7 + 3 = 10 marks)

4. (i) Among eight measurements of the boiling point of a silicon compound, the size of the error was 0.06, 0.04, 0.14, 0.04, 0.11, 0.09, 0.08 and 0.03 Centigrades. Assuming that these data is a random sample from the population given by :

$$f(x;\theta) = \begin{cases} \frac{2(\theta - x)}{\theta^2} & \text{for } 0 < x < \theta \\ 0 & \text{o.w} \end{cases}$$

. Obtain moment estimator of $\theta$.

(ii) The IQ's and 10 teenagers belonging to one ethnic group are 100, 112, 106, 101, 122, 119, 107, 92, 110 and 108, where as those of six teenagers belonging to another ethnic group are 123, 107, 99, 127, 114 and 108. These data are the random samples from normal populations with the means $\mu_1$ and $\mu_2$ and the common variance $\sigma^2$, estimate these parameters by means of the maximum likelihood estimators.

(5 + 5 =10 marks)

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Statistics

## STCP 2.3—PRACTICALS BASED ON STCT 2.3

Time : Two Hours Maximum : 30 Marks

*Answer any* **two** *questions.*
*Each question carries* 15 *marks.*

1. Population of a district(2001) by single years of age is as follows :

| Age | Population | Age | Population | Age | Population | Age | Population |
|---|---|---|---|---|---|---|---|
| 0 | 5750 | 25 | 9191 | 50 | 7191 | 75 | 9160 |
| 1 | 5970 | 26 | 2257 | 51 | 7690 | 76 | 4930 |
| 2 | 1360 | 27 | 8820 | 52 | 8823 | 77 | 3130 |
| 3 | 5625 | 28 | 9021 | 53 | 7840 | 78 | 6910 |
| 4 | 9070 | 29 | 6806 | 54 | 3120 | 79 | 1310 |
| 5 | 6800 | 30 | 10257 | 55 | 3750 | 80 | 7580 |
| 6 | 7875 | 31 | 5270 | 56 | 3499 | 81 | 1544 |
| 7 | 5387 | 32 | 3173 | 57 | 2438 | 82 | 4361 |
| 8 | 2590 | 33 | 9720 | 58 | 5597 | 83 | 1607 |
| 9 | 3791 | 34 | 6738 | 59 | 9106 | 84 | 1391 |
| 10 | 9458 | 35 | 9163 | 60 | 9121 | 85 | 7833 |
| 11 | 3409 | 36 | 1706 | 61 | 10573 | 86 | 1433 |
| 12 | 7934 | 37 | 6586 | 62 | 6410 | 87 | 1004 |
| 13 | 4407 | 38 | 7839 | 63 | 10360 | 88 | 1215 |
| 14 | 4623 | 39 | 3820 | 64 | 8430 | 89 | 4270 |
| 15 | 6134 | 40 | 8797 | 65 | 2497 | 90 | 3973 |
| 16 | 4540 | 41 | 7281 | 66 | 10170 | 91 | 1358 |
| 17 | 2445 | 42 | 8423 | 67 | 10003 | 92 | 1830 |
| 18 | 6896 | 43 | 4577 | 68 | 91187 | 93 | 1226 |
| 19 | 1769 | 44 | 3372 | 69 | 3860 | 94 | 583 |

Turn over

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 20 | 1704 | 45 | 6460 | 70 | 7584 | 95 | 248 |
| 21 | 1756 | 46 | 5144 | 71 | 6420 | 96 | 233 |
| 22 | 4887 | 47 | 5945 | 72 | 7545 | 97 | 184 |
| 23 | 4921 | 48 | 5192 | 73 | 5310 | 98 | 441 |
| 24 | 3998 | 49 | 1801 | 74 | 4530 | 99 | 390 |

Compute Whipple's and Myer's Index.

(15 marks)

2. (A) Compute direct standardized death rate for Kerala using data for India as the standard.

| Age Group | Kerala | | India |
|---|---|---|---|
| | Total Population | ASDR (Per 1000) | Total Population |
| 0-4 | 2759472 | 3.1142 | 108295814 |
| 5-9 | 2851210 | 1.1837 | 112046601 |
| 10-14 | 3053991 | 0.4346 | 99235969 |
| 15-19 | 3106710 | 0.6577 | 82183256 |
| 20-24 | 3045578 | 1.0529 | 74292162 |
| 25-29 | 2653761 | 1.2517 | 68648579 |
| 30-34 | 2208682 | 2.2251 | 59879317 |
| 35-39 | 1931569 | 3.2789 | 51632723 |
| 40-44 | 1579321 | 2.2522 | 43314180 |
| 45-49 | 1284361 | 4.5589 | 36521457 |
| 50-54 | 1095689 | 6.9252 | 29874497 |
| 55-59 | 967916 | 9.7137 | 23926018 |
| 60-64 | 873476 | 15.2415 | 20311173 |
| 65-69 | 685816 | 27.1525 | 14866095 |
| 70 + | 1000966 | 83.7601 | 21274848 |

CDR for Kerala - 6.03.

(B) Compute CBR, GFR, ASFR and TFR using the following information :

| Age Group | Female Population | No. of Births |
|---|---|---|
| 15-19 | 109174 | 17693 |
| 20-24 | 165037 | 52566 |
| 25-29 | 164054 | 43298 |
| 30-34 | 138721 | 23470 |
| 35-39 | 111398 | 10864 |
| 40-44 | 88648 | 4298 |
| 45-49 | 75065 | 1830 |

Total Population is 2450596.

(7 + 8 = 15 marks)

3. Construct abridged life table by using Greville's method for the following data :

| Age Group | ${}_n m_x$ |
|---|---|
| 0-1 | 0.00412 |
| 1-5 | 0.00125 |
| 5-10 | 0.00111 |
| 10-15 | 0.00112 |
| 15-20 | 0.00055 |
| 20-25 | 0.00086 |
| 25-30 | 0.00082 |
| 30-35 | 0.00104 |
| 35-40 | 0.00123 |
| 40-45 | 0.00166 |
| 45-50 | 0.00276 |
| 50-55 | 0.00347 |
| 55-60 | 0.00549 |
| 60-65 | 0.00910 |
| 65-70 | 0.01291 |
| 70-75 | 0.02316 |
| 75-80 | 0.03246 |
| 80-85 | 0.08410 |
| 85-90 | 0.12542 |
| 90-95 | 0.21263 |
| 95-100 | 0.33482 |
| 100+ | 0.38428 |

(15 marks)